STRATEGIES TO INVOKE STUDENTS’ USE OF MATHEMATICAL THINKING POWERS IN ENGINEERING MATHEMATICS

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ABSTRACT

We have been teaching mathematics to engineering undergraduates and our experience has shown that, it was not easy to directly link the topics being taught to the wonders of creation of Allah. In addition, students had certain difficulties with the topics being taught. In general, although our students have demonstrated the ability to answer standard or routine questions, there were still some inconsistencies between their ability to answer questions and their understanding of the concepts and the mathematical procedures that they were using (Mohammad Yusof & Abdul Rahman, 2001). However, using the three frameworks we have brought together, we identified certain components as to make the connection to Islam. In particular, we highlighted the importance of mathematical thinking to be integrated in the teaching and learning of tertiary mathematics. In this presentation, will share how we change our teaching practice so that we could facilitate the development of our students’ ability in understanding the various related mathematical concepts and are able to reconstruct them as parts of a whole. We aimed to support them develop efficient strategies in constructing new knowledge and be empowered with more successful ways of thinking about mathematics. These thinking skills would also benefit the students in various areas of their life. Here, we will share our experience in translating some of the theories into classroom practice and how we being aware of our own personal theory would enhance our preparations of instructions and activities in the classroom.

Keywords: mathematical thinking, engineering mathematics, thinking power
1.0 INTRODUCTION

In this paper, we will be discussing several issues that emerge when we decided to change the way we taught engineering mathematics at the undergraduate level. Our experience in teaching has always been focused on supporting and enhancing students’ understanding of mathematics. However, we felt strongly that we should also contribute to the wholesome development of students’ characters especially our Muslim students in terms of their awareness of their roles in life as khalifatul fil ard. We found that our passion for Mathematics has supported us to develop more flexible thinking, critical and analytical skills. Our dilemma has been how to connect the mathematics taught directly to Islam. We found that this was not always easy as we were teaching engineering mathematics and direct connections were not always found. However, we have found that although our students have some typical difficulties, they have also shown that they have potential capabilities that they could develop. For example, our students have demonstrated the ability to answer standard or routine questions but there were still some inconsistencies between their ability to answer questions and their understanding of the concepts and the mathematical procedures that they were using (Mohammad Yusof & Abdul Rahman, 2001). We also encountered students who found non-routine questions and problem solving difficult. We realised that a more empowering approach is to support students’ development of their own thinking and problem solving skills as well as more independent learning skills. In terms of connecting the course to Islam, there are various components that can be linked but we will focus on the importance of promoting teaching that explicitly supports the skills of thinking and the strategies that we have chosen.

2.0 THINKING AND MUSLIMS

In Arabic, to think = fakkar يَفَكّر - yufakkir يَفَكّر, and in the Quran, the root of the word TAFAKKUR is F-K-R. This root appears in the Quran about 18 times. It has the following meanings: to think upon, consider or examine a thing; to consider a thing in order to obtain a clear knowledge of it; to employ one’s mind, thought or mental consideration upon something; to reflect, think on, ponder over; to consider, contemplate, ponder with care, attention and endeavor, and lastly, arranging of known things (in the mind) in order to attain (the knowledge of) unknown (thing).

Thus, thinking in its various facets, are strongly encouraged and required for Muslims to examine, contemplate and strengthen man’s worship of Allah. Some of the Quranic verses that describe the importance of thinking, contemplation and reflection are:

“Verily, in the creation of the heaven and the earth, and in the alteration of night and day, there are indeed signs for men of understanding-those who remember (the Guidance of) Allah standing, sitting and lying down on their sides, and think & reflect about the creation of the heavens and the earth…” [Aali Imran, 3:190-191]

“And it is He who spread out the earth, and set thereon mountains standing firm and (flowing) rivers: and fruit of every kind He made in pairs, two and two: He draws the night as a veil over the Day. Behold, verily in these things there are Ayaat (proofs, evidences, lessons, signs, etc.) for those who think & reflect! “ [Ar-Rad, 13:3]

It is most unfortunate that, especially in Mathematics, our students have been more accustomed to learning by rote, drill and practice. We found that it was not easy to change their preferences in learning mathematics although there were students who found our
strategies very helpful (Roselainy, 2009). We had identified the mathematical skills that we thought our students needed to become independent learners, particularly, the ability to think flexibly. They should also have the ability to work out and reconstruct ideas from a few examples and be able to reconstruct techniques from a few core ideas. We felt that they should be able to solve non-routine problems or at least make logical and reasonable attempts at solving such problems. However, in changing the way we teach, we realized we had to address firstly, their existing needs, and secondly, on how to ensure their progress through the curriculum. We also needed the students to consider new learning behavior and thus we integrated strategies to encourage students to adopt self-regulated learning behavior.

3.0 ISSUE

In the following sections, we will outline various issues that we had to deal with in changing our teaching practice that arose from our own reflections and concerns that the students’ had expressed about the teaching and learning situations.

Issue 1: Beliefs and attitudes about Mathematics

Findings from research (Mohd. Yusof & Tall, 1994; Yudariah, 1995), showed that our students perceived mathematics as a subject wholly consisting of a conglomeration of facts and procedures. It was also reported that students with a poor track record in mathematics achievements in the past were over anxious when exposed to new problems and concepts. They would give up easily when faced with difficulties and showed great reluctance in persevering with new ideas and techniques. There was significant belief amongst the students that ‘drill & practice’ was the most successful way to master mathematics. So, not surprisingly, many showed resistance in adapting their learning styles and adopting suitable mathematical learning skills more appropriate for learning advanced mathematics. Consequently, they showed some resistance when teaching approaches that required their participation were carried out. In a survey by Khyasudeen et al (1995), 70% of the student respondents claimed to have high motivation towards their learning. However, this was not reflected in their learning behaviour. Responses from the section on study habits showed that they had poor class attendance, did not have complete lecture notes, and did not often participate in class or peer group discussion. Many could not see how the mathematics was relevant to the workplace, let alone how the study of mathematics would help them in strengthening their faith and religious practice in their daily lives. On the other hand, we saw Mathematics as a subject could enhance thinking and problem solving skills, which although presented in such a way to support mathematical learning, would also benefit their ability to think, reflect and solve problems in other areas of their lives. Yudariah Mohd Yusof (1995) found that students’ ability to solve problems can be enhanced through special sessions that taught them mathematical problem solving skills. However, their ability diminished over time if not further supported.

We realized that as teachers, we were concerned about our students’ ability to cope with more advanced mathematics but our students were concerned with being successful in examinations. We felt that students needed to develop their understanding and skills in making sense of new mathematical knowledge but they only wanted to be able to be more competent in using mathematical techniques used in the tests and examination questions! Clearly, we had to address the students’ concerns and at the same time, ensure their progress through the curriculum. How could we convince students that they needed to change the way
they worked with the mathematics? This compelled us to find teaching methods that would maintain a balance between helping students overcome their difficulties, at the same time develop their understanding and skills in making sense of new mathematical knowledge, and enhance their competency in handling mathematical techniques according to the demands of the curriculum as well become more aware of these skills so as to be able to transfer these skills to other areas in their lives.

We were interested in theories that relate to the learning of advanced mathematics, issues that deal with how students learn, how these relate to the instructional design and also we needed to work out how to systematically connect students to Islam. We were not only concerned with developing teaching strategies that would help students overcome their current difficulties but enhances their abilities to cope with more advanced knowledge, become Muslims who are motivated, more aware of their faith and consciously try to improve themselves.

**Issue 2: Developing a Mathematical and Islamic Pedagogy to Support our Ideas**

We looked at various theories of learning and instruction to find pedagogy that would support changes to our practice. Our teaching approach was designed to engage our students on the three levels of the human psyche: behavior, emotions and awareness. We also wanted to help students develop the ability to understand various related mathematical concepts, learn to reconstruct these concepts as parts of a whole and to make the connections between the parts. We believed that through this approach, we could support students during their struggle with mathematical ideas. Thus, using mathematical thinking was an important feature of our lessons. We based our conception of mathematical thinking on the ideas of Mason, Burton & Stacey (1982) but we also used the description of how low and high achievers managed their mathematical knowledge (Gray & Tall, 1994) to determine our content advancement. We used Skemp’s (1993) description of the learning situation to create an environment that would support problem solving. Four processes were identified as central to mathematical thinking, specializing, conjecturing, generalizing and convincing. Specializing is the exploration of meaning by looking at particular cases to make clear some common properties. Conjecturing should naturally follow as a student search for relationships that connects the examples and tries to express and substantiate any underlying patterns. Generalization is the ability to recognize those patterns or regularity and making an attempt in expressing it mathematically. Convincing oneself and then another about the conjecture of the generalization that has been made encourages students to examine their ideas and explicitly communicate it first to themselves and then to others. However, we had to develop our own framework in order to establish connections to Islam.

**Issue 3: Framework for designing classroom instruction and activities**

We found that Watson & Mason’s (1998) ideas on what constituted mathematical thinking powers and mathematical structures were very useful in developing the lesson plans. We used the framework, ‘structure a topic’ (Mason & Johnston-Wilder, 2004) and incorporated various ‘prompts and questions’ (Watson & Mason, 1998) that could invoke students’ use of their own thinking powers. Identifying the thinking processes, for example, specializing and generalizing, ordering and classifying, imagining and expressing, abstracting and instantiating, conjecturing and convincing, were useful in our preparations to provide learning experiences where we could encourage students to use these powers explicitly thus supporting them in recognizing the mathematical processes and structures for themselves. We adapted, modified and created some ‘prompts and questions’ (Watson & Mason, 1998) to
draw students’ attention to the mathematical processes and structures involved in facilitating their understanding of concepts learnt. The use of the prompts and questions would also enable students to guide their own thinking and used as tools to engage with new problems. We felt that we were able to provide students with a vocabulary to master their own thoughts as well as engage in new ones. However, the complete guide of strategies that were used are listed as follows:

a) **Structure of a topic**: we developed a topic by focusing on the mathematics that we wanted our students to learn, in particular, to be able to identify the structures in the mathematical concepts (definitions, facts, theorems, properties, techniques, examples, etc) and made explicit the mathematical powers used (specializing, generalizing, conjecturing, characterizing, organizing, reasoning, etc).

b) **Connecting to Islam**: various components can be used when appropriate, for example, historical contributions of past mathematicians, emphasising the akhlaq of a Muslim towards the pursuit and application of knowledge, and contemplating the wonders of Science.

c) **Structured examples**: examples have always played an important role in teaching but if used purposefully, they enhanced students’ learning. Our approach was to use examples structured in a manner that would lead towards a generality, which we called, ‘structured examples’. Students worked on typical examples first, then on generic examples leading towards more general examples. To further strengthen their understanding and knowledge, non-typical examples were also given and they were asked to make up their own examples. We used students’ own examples to assess what they attended to in the topics taught and helped in uncovering what they did not understand.

d) **Students’ use of their own thinking powers**: the tasks also encouraged students’ use of their own thinking powers. Opportunities for students to explore linkages and connections between mathematical ideas were provided. The tasks allowed students to experience the mathematical thinking activities of specialising and generalising, imagining and anticipating some mathematical relationship, as well as opportunities of expressing mathematical ideas and objects in words, pictures, and symbols. Some opportunities for students to conjecture, reason and work through ideas, were included as an important base for advanced mathematical thinking.

e) **Using mathematical themes; prompts and questions**: we used mathematical themes such as ‘invariance amidst change’, which form the basis for many mathematical theorems and technique; ‘doing and undoing’, which can help students identify features or structures that should be the focus of attention. The ‘prompts and questions’ that we used were specially constructed to make explicit the internal structures of mathematics and mathematical thinking, focused students’ attention to the structures and processes of mathematical thinking, provoked and invoked students’ awareness of their own mathematical thinking powers, and provided students with simple vocabulary to generate mathematical discussion. We started by supporting students in specializing and generalizing. Examples of questions used most often at the beginning were: *What is the same?; What is different?; What can change and what stays the same?; What happens in general?* Some examples of prompts that were used were ‘Give me another example of the same kind’ and ‘Give me a counter-example’.

f) **Active learning**: we provided and created a conducive environment for active learning with an emphasis on student-centered teaching to promote students participation and engagement with the mathematics as well as opportunities for communicating their mathematical knowledge. The important elements that supported effective active learning; talking, listening, reading, writing and reflecting (Meyers &
Jones, 1993); were elements that we actively incorporated in our class activities and tasks. In addition, students were encouraged to work in a group so as to support more discussion amongst students as well as independent learning.

g) Using a workbook: An important consideration was the prevalent learning culture at secondary school, high school and matriculation courses where learning was procedural and achievement was only measured in terms of examination results. Our students were consistently worried about their performance in the examinations and they also needed to see the prompts and questions to become familiar with them. In devising our teaching acts, we took these factors into account and created tasks that would introduce students mathematical ideas, thinking activities as well as provided questions to promote discussions. To ensure students had a compilation of these tasks, we designed a workbook that had five distinctive features to help students to become more familiar with our way of teaching. The five features were different sections in the book such as Illustrations, Structured Examples, Reflections, Review Exercises and Further Exercises.

Issue 4: Self-regulated learning (SRL)

The course is delivered to encourage independent learning and self-regulation. To support and encourage students to adopt SRL processes, the following strategies were used.

a) Forethought and planning: students are given prior information about the course outcomes, weekly schedule, students’ Learning Time calculation and the assessment information, namely, types, dates and marks allocated. In addition, they are also given Learning Guides which gives them information on topics outcomes, amount of time for each topic and sub-topic coverage and the assignment questions. They are encouraged to manage their own learning by using these guides.

b) Performance monitoring: students are given questions to help monitor their knowledge and mathematical skills development. The section is named, Making Sense. The questions in this section is focused on students’ understanding of the concepts taught and their awareness of their facility with the mathematical procedures and techniques.

c) Reflections monitoring: we have identified a section, named Reflections with questions addresses the students’ ability to manage their study, their time, their emotional response to the study of a particular topic or in general, effective learning strategies, learning difficulties, strategies to overcome those difficulties and their levels of motivation. They have to evaluate their strengths, weaknesses, things they like about the course, teaching and learning as well as suggestions of things they would like to change. These self-reflections should influence students’ future planning and goals, initiating the cycle to begin again.

d) Soft skills development: some soft skills were identified to further enhance students’ development. In this course, the skills are;

i. Teamwork: Collaborative and active learning strategies were used to support students’ participation, encourage discussion, and teamworking skills. This will also ensure that they have stamina and encouragement to persevere using the SRL methods.

ii. Communication: since the students mainly work in their groups, much discussion is carried out. Sometimes, a students is called upon to share with the class their understanding of concepts taught and examples of problems that they have solved. In terms of other forms of communication, the sections on
Making Sense and Reflections, address emotional, cognitive and performance monitoring skills which are then shared in written form.

The diverse activities had generated the students’ interest as well as provided them with opportunities to take charge of their learning. At the beginning, students were uncomfortable with the activities as they were different from their usual learning experiences. However, after a few sessions, they became more comfortable with the new environment, showed enthusiasm for group work, sharing of ideas and working out the mathematics for themselves. We believed that the learning environment had facilitated students’ use of their own thinking and increased their communication skills amongst peers and with us. The classes were also livelier. We have seen from the students’ responses (Sabariah et al, 2008) that they became more willing to communicate their difficulties and also to express their understanding of the concepts that were taught. The strategies have also enabled students to start off in doing problems although not necessarily being able to solve them but there was an increased quality of participation and more attempts at communicating with their peers and with us. An important part of our strategy was a section on Reflections where students were asked to reflect on the mathematical concepts as well as the skills that they have have developed. It should be noted, our classes consisted of Muslims and Non-Muslims and thus many of the skills are referred in general. However, personal discussions with Muslim students would refer to the Islamic connections more explicitly. We saw that many students’ written communication was improving progressively. There was also a greater willingness and enthusiasm to do problems, to discuss, explain and share their thinking with others.

4.0 CONCLUSION

We had based our strategies on learning and instruction theories but we found that each learning and instruction theory we studied highlighted certain aspects and obscured others. Thus, we had to evaluate each theory for what it illuminated about learning and how it could guide the development and design of effective instruction most suitable to our needs and concerns. In terms of enriching students’ awareness of Islamic connections, we work out a very broad framework using the basic ideas that supporting students to be better thinkers, reflect and becoming more self-regulated are indeed the connections to husnul khuluq (good akhlaq) which are very Islamic values. In the design and implementation of the teaching strategies, we had to consider the way we taught, the students’ learning environment, the way students learn and work in the classroom as well as how mathematical knowledge grow and develop as well as how personal development can be enriched. More importantly, the whole teaching and learning process must be considered as a cycle of ‘planning, implement, review, modify or change’ and thus, it is a dynamic and continuously improving process where each of us as teachers had to the changing first. Students found changing easier when the learning environment supports and rewards the changes they showed.

REFERENCES


